Finite field-dependent BRS transformation and axial gauges

1998 J. Phys. A: Math. Gen. 31 4217

(http://iopscience.iop.org/0305-4470/31/18/011)

J. Phys. A: Math. Gen. **31** (1998) 4217–4224. Printed in the UK PII: S0305-4470(98)87391-5

Finite field-dependent BRS transformation and axial gauges

Rudra Shyam Bandhu† and Satish D Joglekar‡

Department of Physics, Indian Institute of Technology Kanpur, Kanpur 208016, India

Received 8 September 1997

Abstract. Finite field-dependent BRS (FFBRS) transformation is a generalization of ordinary BRS transformation that can be used to connect actions in different gauge. In this work, we develop the FFBRS transformation that connects the usual Lorentz gauges (with gauge parameter λ) with axial gauges (with gauge parameter λ'). We suggest a possible application of this result to rigorously obtain the prescription of the $1/\eta \cdot q$ type singularity.

1. Introduction

Strong, weak and electromagnetic interactions are very well described by the standard model which is a non-Abelian gauge theory [1]. Practical calculations in non-Abelian gauge theory require a choice of gauge and there are many choices available. Two of the gauges used predominantly are the Lorentz-type gauges and axial-type gauges. Lorentz-type gauges (with gauge-fixing term $-(1/2\lambda)(\partial \cdot A)^2$) have the natural advantage of simplicity of Feynman rules, covariance and the possibility of checking the gauge independence of results by studying the dependence on λ [2]. There are also no ambiguities in dealing with these gauges when dealing with the singularities of the propagators. Naturally a large number of practical as well as formal calculations have been done in Lorentz gauges. A disadvantage of Lorentz gauges in non-Abelian gauge theories is, however, that they require a ghost action; and this complicates calculations, operator product expansions [3] etc. For this reason, another set of gauges has often found favour in calculations; namely, the axial gauges $\eta \cdot A = 0$, η_μ being a set of four real numbers. These gauges have the formal advantage that the ghost term is trivial (free) and consequently calculations are simplified considerably in these gauges [4]. In fact, first practical calculations in quantumchromodynamics (QCD) were done in these gauges [5]. These gauges are, however, accompanied by a lack of explicit covariance. More importantly, they contain in their propagators singularities of the form $1/((\eta \cdot q)^p)$ which need to be carefully interpreted if calculations are to be done for Feynman diagrams involving loops.

Various prescriptions have been proposed for this $1/(\eta \cdot q)$ singularity. Two of these are the 'principle value prescription' (PVP) [6] and the 'Mandelstam–Leibbrandt prescription' (MLP) [7]. Both of these are *ad hoc* prescriptions and lead to a variety of problems. While PVP violates the Ward–Takahashi (WT) identity to one loop order for $\eta^2 = 0$ and has several other difficulties associated with it [8], the MLP in the light cone gauge ($\eta^2 = 0$)

† Present address: Department of Physics, Ohio State University, Columbus, OH 43210, USA.
‡ E-mail address: sfj@iitk.ernet.in

0305-4470/98/184217+08$19.50

leads to Lorentz-non-invariant integrals or non-local counterterms [9]. A prescription has also been derived for gauges of the type $A_1+\lambda A_3=0$ by Landshoff and von Niewenhuizen [10] using canonical quantization. For further attempts see [11].

We want to approach the two set of gauges differently. As calculations in Lorentz gauges are unambiguous, a similar unambiguous procedure for axial gauges could possibly be arrived at if one were to establish a link between the two gauges. In fact, a novel field transformation, the 'finite field-dependent BRS (FFBRS) transformation', has recently been proposed [12] with just a view to connecting generating functionals in different gauges and has successfully been applied in a number of cases there. Our aim is to generalize the work of [12] to establish the FFBRS transformation that connects Lorentz-type gauges to axial-type gauges. Such a field transformation should enable one to go back and forth between the two sets of gauges. In this work, we mainly restrict ourselves to this problem, making comments on any applications at the end. We hope to report these applications elsewhere.

We remark that interest in connecting these sets of gauges is more than for formal reasons alone. Discrepancies have been reported among the calculations for anomalous dimensions of operators (which could have observable effects) in the two sets of gauges [13]. Our FFBRS transformation, explicitly constructed (albeit complicated) connects these sets of gauges, we believe, for the first time. We hope to exploit the transformation in this problem in the future.

2. Summary of results on FFBRS transformations

The familiar BRS transformations for gauge theory with Faddeev–Popov effective action (FPEA) are

$$\begin{aligned}
\delta A_\mu^\alpha &= (D_\mu c)^\alpha \delta\Lambda \\
\delta c^\alpha &= -\frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma \delta\Lambda \\
\delta \bar{c}^\alpha &= -\frac{\partial\cdot A^\alpha}{\lambda}\delta\Lambda
\end{aligned}\tag{2.1}$$

where $\delta\Lambda$ is an anticommuting infinitesimal constant. It was observed by Joglekar and Mandal [12] that $\delta\Lambda$ need not be infinitesimal, nor need it be field-independent for (2.1) to be a symmetry of the FPEA as long as it does not depend explicitly on x^μ. Thus, the following transformations

$$\begin{aligned}
A'^\alpha_\mu(x) &= A^\alpha_\mu(x) + D^{\alpha\beta}_\mu c^\beta(x)\Theta[\phi] \\
c'^\alpha(x) &= c^\alpha(x) - \tfrac{1}{2} g f^{\alpha\beta\gamma} c^\beta(x) c^\gamma(x)\Theta[\varphi] \\
\bar{c}'^\alpha(x) &= \bar{c}^\alpha(x) - \frac{\partial\cdot A^\alpha(x)}{\lambda}\Theta[\phi]
\end{aligned}\tag{2.2}$$

or generically

$$\phi'_i(x) = \phi_i - \delta_{\mathrm{BRS}}[\phi_i]\Theta[\phi] \tag{2.2a}$$

where $\Theta[\phi]$ is an x-independent functional of fields A, c and $\bar{c}$ (generically denoted by ϕ_i), are also a symmetry of the effective action and will be called 'finite field-dependent BRS (FFBRS) transformations' for obvious reasons.

Transformations of the form (2.2) were used, in [12], to connect the actions of different kinds for Yang–Mills theory. The FPEA is invariant under (2.2) but the functional measure is not, and the Jacobian for the FFBRS transformations (in special cases dealt with in [12]) can be expressed effectively as $\exp[\mathrm{i}S_1]$; S_1 then explains the difference between the effective actions for different formulations of gauge theories. Thus FFBRS transformations that connect the usual FPEA S_{eff} in linear gauges with gauge parameter λ to (i) the most general BRS–anti-BRS symmetric action in linear gauges, (ii) the FPEA in quadratic gauges and (iii) the FPEA with another distinct gauge parameter λ' were explicitly constructed in [12].

In this work we shall carry out this construction for connecting Lorentz-type and axial-type gauges following the general algorithm presented in [12]. The general procedure for constructing the FFBRS transformation of (2.2) for a given case is outlined below.

Let us denote by $\phi_i(x)$ the fields A, c and $\bar{c}$ generically. We then construct a transformation from these to A', c' and $\bar{c}'$ (generically denoted by $\phi_i'(x)$) by a continuous interpolation. We consider the intermediate fields $\phi_i(x,\kappa)$ $(0 \leqslant \kappa \leqslant 1)$ satisfying the infinitesimal field-dependent BRS transformations

$$\frac{\mathrm{d}\phi_i(x,\kappa)}{\mathrm{d}\kappa} = \delta_{\text{BRS}}[\phi_i(x,\kappa)]\Theta'[\phi(x,\kappa)] \tag{2.3}$$

where $\Theta'[\phi(x,\kappa)]$ is a κ-independent, but a local functional of ϕ_i; as yet unspecified. These can then be integrated to yield (2.2) (for certain special cases of $\Theta[\varphi]$) where we identify $\phi'(x) = \phi(x, \kappa = 1)$. (The relation between Θ and Θ' is reproduced later in (2.9).)

The Jacobian for such transformations is

$$DADcD\bar{c} = J(\kappa)DA(\kappa)Dc(\kappa)D\bar{c}(\kappa) \tag{2.4}$$

and can be replaced (within the functional integral) as

$$J(\kappa) \to \exp[\mathrm{i}S_1[\varphi(x,\kappa)]] \tag{2.5}$$

with a certain *local* $S_1[\varphi]$ in certain cases of $\Theta'[\varphi]$; the condition for existence of S_1 is

$$\int D\varphi(x)\left[\frac{1}{J}\frac{\mathrm{d}J}{\mathrm{d}\kappa} - \mathrm{i}\frac{\mathrm{d}S_1[\varphi(x,\kappa)]}{\mathrm{d}\kappa}\right]\exp[\mathrm{i}(S_{\text{eff}} + S_1)] = 0 \tag{2.6}$$

then,

$$W = \int D\varphi \exp[\mathrm{i}S_{\text{eff}}[\varphi]] = \int D\varphi' \exp[\mathrm{i}S_{\text{eff}}[\varphi'] + \mathrm{i}S_1[\varphi']]. \tag{2.7}$$

Thus, to summarize, if a local $\Theta'[\varphi]$ can be found and if a local action $S_1[\varphi]$ can be found such that the Jacobian for (2.3), J, satisfies equation (2.6), then the integrated version of (2.3), viz (2.2), (with Θ related to Θ') takes us from W with FPEA S_{eff}

$$W = \int D\varphi \exp[\mathrm{i}S_{\text{eff}}[\varphi]] \tag{2.8a}$$

to that with net effective action $S_{\text{eff}}[\varphi'] + S_1[\varphi']$, viz

$$W' = \int D\varphi' \exp[\mathrm{i}S_{\text{eff}}[\varphi'] + \mathrm{i}S_1[\varphi']] \tag{2.8b}$$

W and W' being numerically equal.

Now, to connect Lorentz-type gauges to axial-type gauges, S_1, the difference between effective actions, being known (see the next section), it is then a matter of conjecturing Θ' and showing that (2.6) is satisfied by the Jacobian. Once Θ' is so obtained, the FFBRS transformations are completely determined by (2.2) with $\Theta[\varphi]$ given by

$$\Theta[\varphi] = \Theta'[\varphi]\frac{\exp[f[\varphi]] - 1}{f[\varphi]} \tag{2.9}$$

where $f[\varphi]$ is determined by the BRS variation of $\Theta[\varphi]$

$$f[\varphi] = \sum_i \int \mathrm{d}^4 x \frac{\delta\Theta'[\varphi]}{\delta\varphi_i(x)} \delta_{\mathrm{BRS}}\varphi_i(x) \tag{2.10}$$

where the sum over i goes over A, c, $\bar{c}$.

(It may be remarked that the infinitesimal gauge transformation that takes one from S_{eff} of the Lorentz gauges 'towards' the S_{eff} of the axial gauges can be used to guess the form for Θ', so that Θ' is not entirely left to arbitrary guess work. More on this in the next section.)

3. Construction of FFBRS transformation

In this section we carry out the programme mentioned in the last section, of constructing FFBRS transformations that connect explicitly the Lorentz-type gauges with the axial-type gauges.

The Lorentz-type gauges with the free gauge parameter λ are described by the FPEA

$$S^L_{\mathrm{eff}} = \int \mathrm{d}^n x \mathcal{L}^L_{\mathrm{eff}}[A, c, \bar{c}] \tag{3.1}$$

with

$$\mathcal{L}^L_{\mathrm{eff}} = \mathcal{L}_0 - \frac{1}{2\lambda} \sum_\alpha (\partial \cdot A^\alpha)^2 - \bar{c}^\alpha M^{\alpha\beta} c^\beta \tag{3.2}$$

where

$$\mathcal{L}_0 = -\tfrac{1}{4} F^\alpha_{\mu\nu} F^{\alpha\mu\nu} \tag{3.3}$$

and

$$\bar{c}^\alpha M_{\alpha\beta} c_\beta \equiv \bar{c}^\alpha \partial^\mu D^{\alpha\beta}_\mu c^\beta. \tag{3.4}$$

The axial-type gauges with the free gauge parameter λ are described by the effective action

$$S^A_{\mathrm{eff}} = \int \mathrm{d}^n x \mathcal{L}^A_{\mathrm{eff}}[A, c, \bar{c}] \tag{3.5}$$

with

$$\mathcal{L}^A_{\mathrm{eff}} = \mathcal{L}_0 - \frac{1}{2\lambda} \sum_\alpha (\eta \cdot A)^2 - \bar{c}^\alpha \tilde{M}^{\alpha\beta} c^\beta \tag{3.6}$$

where

$$\tilde{M}^{\alpha\beta} = \eta^\mu D^{\alpha\beta}_\mu [A]. \tag{3.7}$$

It is in the limit where $\lambda \to 0$ in (3.6) that one recovers the formally ghost-free gauge

$$\eta \cdot A^\alpha = 0.$$

We next need to construct an FFBRS transformation connecting (3.2) and (3.6). For this, we construct an interpolating effective Lagrangian for the mixed gauge condition; with gauge functional

$$F^\alpha[A] = (1 - \beta)\partial \cdot A^\alpha + \beta\eta \cdot A^\alpha. \tag{3.8}$$

The Faddeev–Popov effective action for this gauge is

$$\mathcal{L}^M_{\mathrm{eff}} = \mathcal{L}_0 - \frac{1}{2\lambda} \sum_\alpha [(1 - \beta)\partial \cdot A^\alpha + \beta\eta \cdot A^\alpha]^2 - \bar{c}[(1 - \beta)M + \beta\tilde{M}]c \tag{3.9}$$

where for $\beta = 0$, one obtains $\mathcal{L}^L_{\text{eff}}$ in Lorentz-type gauges and for $\beta = 1$, one obtains $\mathcal{L}^A_{\text{eff}}$ in axial type-gauges.

The ansatz for Θ' of the infinitesimal field-dependent BRS of equation (2.3) is obtained by consideration of the infinitesimal *gauge* transformation that takes one from $\beta = 0$ (Lorentz gauges) to $\beta = \Delta\beta$ (a small admixture of the axial term). This is known, from the general result, to be [2]

$$\begin{aligned}\delta A^\alpha_\mu &= D^{\alpha\beta}_\mu M^{-1}_{\beta\gamma}\Delta F^\gamma\,\text{(summation-integration convention used)}\\ &\equiv D^{\alpha\beta}_\mu M^{-1}_{\beta\gamma}\Delta\beta(\eta\cdot A^\gamma - \partial\cdot A^\gamma)\end{aligned} \tag{3.10}$$

and this suggests that we consider the following FFBRS transformation with

$$\Theta' = \mathrm{i}\gamma\int \mathrm{d}^n y\,\bar{c}^\gamma(y)(\partial\cdot A^\gamma - \eta\cdot A^\gamma)(y)$$

viz

$$\begin{aligned}\delta A^\alpha_\mu(x) &= \mathrm{i}\gamma D^{\alpha\beta}_\mu c^\beta(x)\int \mathrm{d}^n y\,\bar{c}^\gamma(y)(\partial\cdot A^\gamma - \eta\cdot A^\gamma)(y)\\ \delta c^\alpha(x) &= -\mathrm{i}\gamma\frac{1}{2}gf^{\alpha\beta\delta}c^\beta(x)c^\delta(x)\int \mathrm{d}^n y\,\bar{c}^\gamma(y)(\partial\cdot A^\gamma - \eta\cdot A^\gamma)(y)\\ \delta\bar{c}^\alpha(x) &= -\mathrm{i}\gamma\frac{\partial\cdot A}{\lambda}\int \mathrm{d}^n y\,\bar{c}^\gamma(y)(\partial\cdot A^\gamma - \eta\cdot A^\gamma)(y).\end{aligned} \tag{3.11}$$

(We note that the first equation of (3.11) upon replacement of $c_\beta(x)\bar{c}_\gamma(y) \to \mathrm{i}M^{-1}_{\beta\gamma}(x, y)$ would go into (3.10) if γ is identified with $-\Delta\beta$. Such a relationship is used in *suggesting* the form of FFBRS transformation (3.11), which will now be *proved* to work.)

Equations (3.11) are written in brief as

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\kappa}\phi_i(x,\kappa) &= [\delta_{\text{BRS}}\phi_i(x,\kappa)]\mathrm{i}\gamma\int \mathrm{d}^n y\,\bar{c}^\gamma(y,\kappa)[\partial\cdot A^\gamma(y,\kappa) - \eta\cdot A^\gamma(y,\kappa)]\\ &= [\delta_{\text{BRS}}\phi_i(x,\kappa)]\Theta'[\varphi(x,\kappa)].\end{aligned} \tag{3.12}$$

These define a one-parameter family of fields $A(x,\kappa)$, $c(x,\kappa)$ and $\bar{c}(x,\kappa)$. In terms of these we construct $W[\kappa]$

$$W[\kappa] \equiv \int D\varphi(\kappa)\,\mathrm{e}^{\mathrm{i}[S_{\text{eff}}[\varphi(\kappa)] + S_1[\varphi(\kappa),\kappa]]} \tag{3.13}$$

such that:

$$W[\kappa] \text{ is independent of } \kappa\text{:} \qquad \frac{\mathrm{d}W}{\mathrm{d}\kappa} = 0 \tag{3.14a}$$

$$S_1[\varphi(\kappa),\kappa]|_{\kappa=0} = 0 \tag{3.14b}$$

$$W[0] = \int D\varphi(0)\mathrm{e}^{\mathrm{i}[S^L_{\text{eff}}[\varphi(0)]} \tag{3.14c}$$

$$W[1] = \int D\varphi(1)\mathrm{e}^{\mathrm{i}[S^A_{\text{eff}}[\varphi(1)]}. \tag{3.14d}$$

In other words, the transformation (3.12) goes from $\phi(0)$ to $\phi(1)$ such that $W[0]$ of (3.14*c*) is transformed *in form* into $W[1]$ (numerically unaltered) of (3.14*d*) which involves the axial-gauge type action S^A_{eff}.

From (3.13) and (3.14*d*), one can read

$$S_1[\varphi(\kappa),\kappa]|_{\kappa=1} = [S^A_{\text{eff}}[\varphi(1)] - [S^L_{\text{eff}}[\varphi(1)] = \frac{1}{2\lambda}(\partial\cdot A^{\alpha'})^2 - \frac{1}{2\lambda}(\eta\cdot A^{\alpha'})^2 - \bar{c}'\tilde{M}'c' + \bar{c}'M'c'. \tag{3.15}$$

Looking at the form of (3.15) and the kind of terms that are present in the interpolating Lagrangian $\mathcal{L}^M$ of (3.9), we postulate the following form for S_1

$$S_1[\varphi(\kappa),\kappa] = \xi_1(\kappa)(\partial\cdot A)^2 + \xi_2(\kappa)(\eta\cdot A)^2 + \xi_3(\kappa)(\partial\cdot A)(\eta\cdot A) + \xi_4(\kappa)\bar{c}Mc + \xi_5(\kappa)\bar{c}\tilde{M}c \tag{3.16}$$

(all fields here are functions of $\kappa : A = A(x,\kappa)$ etc). (3.14*b*) and (3.15) then imply the following constraints

$$\xi_i(0) = 0 \qquad i = 1, 2, \ldots, 5 \tag{3.17}$$

$$\xi_1(1) = \frac{1}{2\lambda} = -\xi_2(1) \qquad \xi_3(1) = 0 \qquad \xi_4(1) = \xi_5(1) = 1. \tag{3.18}$$

Now, we want to impose equation (2.6) on the assumed forms for Θ' (of (3.12)) and S_1 (of (3.16)). Equation (2.6) is valid for an arbitrary κ ($0 \leqslant \kappa \leqslant 1$). In this regard, we note that $-(1/J)(\mathrm{d}J/\mathrm{d}\kappa)\,\mathrm{d}\kappa$ is the Jacobian for the infinitesimal transformations of equation (3.12) and receives a contribution from the non-local dependences of $\delta\varphi_i(\kappa)$ contained in Θ'. The result is

$$\frac{1}{J}\frac{\mathrm{d}J}{\mathrm{d}\kappa} = -\mathrm{i}\gamma\int \mathrm{d}^n x\left[\bar{c}^\alpha(x)(\partial_\mu - \eta_\mu)D_\mu^{\alpha\beta}c^\beta(x) + \frac{\partial\cdot A^\alpha}{\lambda}(\partial\cdot A^\alpha - \eta\cdot A^\alpha)\right]. \tag{3.19}$$

We also note that in evaluating $\mathrm{d}S_1/\mathrm{d}\kappa$ we not only differentiate the explicit dependence of S_1 on κ through ξ but also the implicit dependence on κ through its dependence on $\phi_i(\xi,\kappa)$. Condition (2.6) then reads

$$\begin{aligned}\int D\varphi\exp[\mathrm{i}(S_{\mathrm{eff}}^L + S_1)] \times \int \mathrm{d}^4x\bigg\{ & Mc\Theta'\left[\partial\cdot A\left(2\xi_1 - \frac{\xi_4}{\lambda}\right) + \eta\cdot A\xi_3\right] \\ & + \tilde{M}c\Theta'\left[\partial\cdot A\left(\xi_3 - \frac{\xi_5}{\lambda}\right) + 2\eta\cdot A\xi_3\right] + (\partial\cdot A)^2\left(\xi_1' - \frac{\gamma}{\lambda}\right) \\ & + (\eta\cdot A)^2(\xi_2') + \partial\cdot A^\alpha\eta\cdot A^\alpha\left(\xi_3' + \frac{\gamma}{\lambda}\right) \\ & + \bar{c}Mc(\xi_4' - \gamma) + \bar{c}\tilde{M}c(\xi_5' + \gamma)\bigg\} = 0.\end{aligned} \tag{3.20}$$

The last two terms in the integrand of (3.20) are dependent on $\bar{c}$ in a local fashion. The contribution of these terms can possibly vanish by the antighost equation of motion. This can only happen if the ratio of coefficients of the two terms is identical to the ratio of coefficients of $\bar{c}Mc$ and $\bar{c}\tilde{M}c$ in $S_{\mathrm{eff}}^L + S_1$. (Here dimensional regularization in which $\delta^n(0)$ terms can be dropped has been assumed.) This requires that

$$\frac{\xi_4' - \gamma}{\xi_4 - 1} = \frac{\xi_5' + \gamma}{\xi_5}. \tag{3.21a}$$

Now, among the remaining terms the Θ' dependent terms can (possibly) be converted into local terms by the antighost equation of motion (if this cannot be done, they remain non-local and cannot be cancelled). This can only work if the two Θ' dependent terms combine in a certain manner, depending again on the ratio of coefficients of $\bar{c}Mc$ and $\bar{c}\tilde{M}c$ in terms in $S_{\mathrm{eff}}^L + S_1$. This requires that

$$\frac{2\xi_1' - \xi_4/\lambda}{\xi_4 - 1} = \frac{\xi_3 - \xi_5/\lambda}{\xi_5} \tag{3.21b}$$

$$\frac{\xi_3}{\xi_4 - 1} = \frac{2\xi_2}{\xi_5} \tag{3.21c}$$

so that when (3.21*b*) and (3.21*c*) are fulfilled, the Θ' dependent terms get converted to local terms of the form of the remaining terms ($(\partial \cdot A)^2$, $(\eta \cdot A)^2$, $\partial \cdot A \eta \cdot A$ type). We now require that the coefficient of these terms in this combination vanish and this leads to three further constraints

$$\xi_1' - \frac{\gamma}{\lambda} - \frac{\gamma(2\xi_1 - \xi_4/\lambda)}{\xi_4 - 1} = 0 \tag{3.21d}$$

$$\xi_2' - \frac{\gamma \xi_3}{\xi_4 - 1} = 0 \tag{3.21e}$$

$$\xi_3' + \frac{\gamma}{\lambda} - \frac{\gamma(\xi_3 - 2\xi_1 + \xi_4/\lambda)}{\xi_4 - 1} = 0. \tag{3.21f}$$

Thus equations (3.21*a*–*f*) must be solved for the five functions $\xi_i(\kappa)$ ($i = 1, 2, \ldots, 5$) and γ with the initial *and* final constraints (3.17) and (3.18). Any solution of this set will constitute a desired solution to the problem. We shall seek a special solution in which the ghost terms for *any* κ take the form present in $\mathcal{L}^M_{\text{eff}}$ of (3.9), i.e.

$$-\bar{c}(1 - \beta(\kappa))Mc - \bar{c}\beta(\kappa)\tilde{M}c. \tag{3.22}$$

This requires that $\xi_4(\kappa) = \beta(\kappa) = -\xi_5(\kappa)$, so that $\xi_4 + \xi_5 = 0$; hence

$$\xi_4' + \xi_5' = 0. \tag{3.21g}$$

The solution of (3.21*a*–*g*) is easy to obtain and has a simple form; we state the result directly

$$\begin{aligned} \xi_1(\kappa) &= \frac{1}{2\lambda}[1 - (1 - \kappa)^2] \\ \xi_2(\kappa) &= -\frac{\kappa^2}{2\lambda} \\ \xi_3(\kappa) &= \kappa(\kappa - 1)/\lambda \\ \xi_4(\kappa) &= \kappa = -\xi_5(\kappa) \\ \gamma &= 1. \end{aligned} \tag{3.23}$$

To summarize, we have constructed a field transformation $\phi_i(x) \rightarrow \phi_i'(x)$ by interpolation via $\phi_{\text{i}}(x, \kappa)$, $0 \leqslant x \leqslant 1$, such that (i) we leave $S^L_{\text{eff}}[\varphi]$ invariant and (ii) we take $W = \langle 0|0\rangle$ of Lorentz-type gauges given by (2.7) into $W(\kappa)$

$$W(\kappa) = \int D\varphi(\kappa) \mathrm{e}^{\mathrm{i}S^L_{\text{eff}}[\varphi(\kappa)] + \mathrm{i}S_1[\varphi(\kappa)]} \equiv W \tag{3.24}$$

such that for $\kappa = 0$ we have, in the exponent, S^L_{eff} of Lorentz-type gauges, for $0 < \kappa < 1$ we have the FPEA of mixed-type gauges S^M_{eff} of equation (3.9) and for $\kappa = 1$ we have the FPEA of axial-type gauges. This field transformation is given by the FFBRS transformation of (2.2) where $\Theta[\varphi]$ is obtained via $\Theta'[\varphi]$ of equation (3.12) (with $\gamma = 1$) via equation (2.9).

4. Conclusions and further directions

In conclusion, we have formally constructed a field transformation that connects Lorentz-type to axial-type gauges. The field transformation, though involved in form, can, we believe, be put to a number of uses. One of these, which we propose to report elsewhere, is to obtain a rigorous prescription for the $1/\eta \cdot q$ singularity in the propagator for the axial gauge. We know that the $1/q^2$ singularity in Lorentz-type gauges is handled simply by replacing $q^2 \rightarrow q^2 + \mathrm{i}\epsilon$. This effectively is done by adding a term $-\mathrm{i}\epsilon A^\alpha_\mu A^{\alpha\mu} + \mathrm{i}\epsilon\bar{c}c$ to S^L_{eff}

of the Lorentz-type gauges. We then propose to apply the FFBRS transformation (suitably truncated to the present context) to this *net* effective action and from this obtain the implied modification in $S^{A}_{\rm eff}$, the axial-gauge effective action. This will then enable us to know how the $(\eta \cdot q)^{-1}$ singularity is to be dealt with. This rather involved calculation will be reported separately.

Another possible application is to the resolution of the discrepancy reported in [13] between axial and Lorentz gauges. We expect several other applications of our results.

References

[1] Cheng T-P and Li L-F 1984 *Gauge Theory of Elementary Particle Physics* (Oxford: Clarendon)
[2] Abers E and Lee B W 1973 *Phys. Rep.* C **9** 1
[3] Joglekar S D and Lee B W 1976 *Ann. Phys.* **97** 160
Joglekar S D 1977 *Ann. Phys.* **108** 233
Joglekar S D 1978 *Ann. Phys.* **109** 210
[4] Konetschny W and Kummer W 1976 *Nucl. Phys.* B **108** 397
[5] Gross D and Wilkzek F 1974 *Phys. Rev.* D **9** 980
[6] Kummer W 1975 *Nucl. Phys.* B **100** 106
[7] Leibbrandt G 1984 *Phys. Rev.* D **29** 1699
Mandelstam S 1983 *Nucl. Phys.* B **213** 149
[8] For difficulties with PVP see e.g. references in Leibbrandt G 1987 *Rev. Mod. Phys.* **59** 1067
[9] Kreuzer M *et al* 1987 *Phys. Lett.* **196B** 557
Pollock G 1989 *Phys. Rev.* D **40** 2027
[10] Landshoff P V and von Niewenhuizen P 1994 *Phys. Rev.* D **50** 4157
[11] Park D K, New prescription in lightcone gauge theories *Preprint* hep-th/9509010
[12] Joglekar S D and Mandal B P 1995 *Phys. Rev.* D **51** 1919
[13] Hamburg R and van Neervan W 1992 *Nucl. Phys.* B **379** 143